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Summary

Charged particle optics has traditionally been based on the concept of midplane symmetry. Accelerators, beamlines, and spectrometers are often midplane symmetric in their entirety. Departures from midplane symmetry are usually obtained by rotating midplane symmetric components about a longitudinal axis.

Misalignments and magnet imperfections introduce non midplane symmetric components into a beam line. Vertical steering and correction of the effect of errors can be accomplished through the deliberate introduction of midplane symmetry violating components in bending magnets. We have worked out the equations of motion and derived the transfer matrix for bending magnets with small midplane symmetry violating field components. The solutions are linearized in the non midplane symmetric magnetic field components.

Introduction

The position and direction of motion of a particle at any point in a beam line can be expressed in terms of a six-component vector X, where

$$X = \begin{pmatrix} x \\ x' \\ y \\ y' \\ \ell \\ \delta \end{pmatrix} \qquad (1)$$

This vector is measured with respect to an assumed reference trajectory. The reference trajectory is taken to be the path of a single specific particle whose momentum is the central design momentum of the optical system. The spatial coordinates x and y measure the transverse distance of a particle from the reference trajectory. The component x is in the horizontal (bend) plane and y is in the vertical. Their derivatives x' and y' are with respect to distance along the reference trajectory. The coordinate ℓ represents the longitudinal separation between an arbitrary particle and the reference particle, when the two particles start at the beginning of the system at the same time. Finally, there is the fractional momentum deviation δ from the reference particle.

The components of the vector at any point in the beam line can be expressed as functions of the components of the initial vector. If we represent this function as a power series in the six variables we get

$$X_1 = RX_o + TX_oX_o + UX_oX_oX_o \qquad (2)$$

The elements of the matrix R are referred to as being of first order; T is second order; U is third, etc.

Midplane symmetry requires that some of the matrix elements must equal zero. The first-order matrix is then block diagonalized so that motion in the x and y planes are independent. The fractional momentum deviation δ affects only horizontal motion, and ℓ is affected by only x, x', and δ. The columns of the first-order transfer matrix are known as characteristic rays. Their nonzero components (excluding those for ℓ) are given special names, so that

$$\begin{aligned} (x|x_o) &= c_x(s) & (x'|x_o) &= c_x'(s) \\ (x|x_o') &= s_x(s) & (x'|x_o') &= s_x'(s) \\ (x|\delta) &= d_x(s) & (x'|\delta) &= d_x'(s) \\ (y|y_o) &= c_y(s) & (y'|y_o) &= c_y'(s) \\ (y|y_o') &= s_y(s) & (y'|y_o') &= s_y'(s) \end{aligned} \qquad (3)$$

For a second-, third-, or higher-order matrix element to be nonzero, the vertical coordinates must appear an even number of times in its notation. Thus the second-order matrix elements $(x|x_o^2)$, $(x|y_oy_o')$, $(x|x_o\delta)$, $(y|x_oy_o)$, and $(y|y_o\delta)$ may be nonzero. The elements $(x|x_oy_o)$, $(x|y_o\delta)$, and $(y|y_oy_o')$ are required by midplane symmetry to be zero.

We now consider small violations of midplane symmetry. The symmetry violating magnetic fields are considered to be error fields, so that the reference trajectory remains unchanged. Now, however, there is a vertically bending magnetic field component, so that a charged particle with the reference momentum which initially follows the reference trajectory cannot continue to follow the reference trajectory. Equation (3) then acquires a zero'th order term and becomes

$$X_1 = X_{1s} + RX_o + TX_oX_o + UX_oX_oX_o \qquad (4)$$

The zero'th order term X_{1s} is the distance by which the particle initially on the reference trajectory deviates from the reference trajectory at the magnet exit.

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There is also now coupling between planes so that many matrix elements which were zero in the case of midplane symmetry, acquire nonzero values. Specifically, the upper left of R is now a complete four-by-four matrix with no block diagonalization. Second-order terms such as $(x|x_0y_0)$, $(x|y_0\delta)$, and $(y|y_0y_0')$ acquire nonzero values. The only remaining zero values result from the fact that, in a passive magnetic system, the transverse coordinates and δ are not affected by ℓ, and the transverse coordinates and ℓ do not affect δ.

Magnetic Field Expansion

We now specialize to the case of a bending magnet where the field strength is a function of only the transverse coordinates. By Maxwell's equations, the magnetic field is completely specified if its components are given in the magnetic midplane. Midplane symmetry requires that the horizontal magnetic field (B_x) be zero on the magnetic midplane. The vertical field component (B_y) is then given by

$$B_y(x,0,s) = B_0(1 - nhx + \beta h^2x^2 + \dots) \qquad (5)$$

Here h is the curvature of the reference trajectory, equal to the reciprocal of the radius of curvature.

Violation of midplane symmetry allows nonzero values of B_x on the magnetic midplane. The horizontal field component can then be given an expansion similar to that of the vertical component.

$$B_x(x,0,s) = B_0(v_R - n'hx + \beta'h^2x^2 + \dots) \qquad (6)$$

Here the primes do not indicate derivatives, but rather that n' and β' are different quantities from n and β. To avoid confusion we will write the differential equations of motion without use of primes to indicate differentiation.

Solving Maxwell's equations in a curvilinear coordinate system, the complete magnetic field expansions become

$$B_x = B_0[v_R - hn'x - hny + h^2\beta'x^2 + 2h^2\beta xy - \frac{1}{2}h^2(2\beta'-n'-v_R)y^2] \qquad (7)$$

$$B_y = B_0[1 - hnx + h(n'-v_R)y + h^2\beta x^2 - h^2(2\beta'-n'-v_R)xy - \frac{1}{2}h^2(2\beta-n)y^2]$$

Zero'th and First Order Solution

The first order equations of motion now become

$$\frac{d^2x}{ds^2} + (1-n)h^2x = h^2(v_R - n')y + h\delta \qquad (8)$$

$$\frac{d^2y}{ds^2} + nh^2y = h^2(2v_R-n')x + v_Rh - v_Rh\delta$$

There are two obvious differences from the midplane symmetric case. The first is the mixing of planes, where there is a term in y on the right side of the equation for x and visa versa. The second is the effect of the vertically bending field on the right side of the second equation. This vertically bending field produces a trajectory shift and an associated vertical dispersion.

At this point, we are faced with a choice of two approaches. The two equations can be solved exactly by finding eigenvectors in the transverse coordinates where the equations separate. Alternatively, we can consider the midplane symmetry violating terms to be small and solve for their effect as a perturbation of the midplane symmetric solution.

We choose the latter approach since midplane symmetry violating magnetic field components are invariably significantly smaller than the midplane symmetry respecting components. The eigenvector approach is also noticeably more complicated in first order and prohibitive in second order.

A first order perturbative solution using Green's functions yields:

$$y_{1S} = v_R\frac{1-c_y}{nh} \qquad (9)$$

$$y'_{1S} = v_Rhs_y$$

$$(x|y_0) = (v_R-n')\frac{c_y-c_x}{1-2n}$$

$$(x|y'_0) = (v_R-n')\frac{s_y-s_x}{1-2n}$$

$$(x'|y_0) = (v_R-n')\frac{c'_y-c'_x}{1-2n}$$

$$(x'|y'_0) = (v_R-n')\frac{s'_y-s'_x}{1-2n}$$

$$(y|x_0) = (n'-2v_R)\frac{c_x-c_y}{1-2n}$$

$$(y|x'_0) = (n'-2v_R)\frac{s_x-s_y}{1-2n}$$

$$(y|\delta) = -v_R d_y + (n'-2v_R)\frac{d_x-d_y}{1-2n}$$

$$(y'|x_0) = (n'-2v_R)\frac{c'_x-c'_y}{1-2n}$$

$$(y'|x'_0) = (n'-2v_R)\frac{s'_x-s'_y}{1-2n}$$

$$(y'|\delta) = -v_R d'_y + (n'-2v_R)\frac{d'_x-d'_y}{1-2n}$$

$$(\ell|y_0) = (v_R-n')\frac{d'_y-d'_x}{1-2n}$$

$$(\ell|y'_0) = (v_R-n')\frac{d_y-d_x}{1-2n}$$

Second-Order Terms

The equations of motion expanded to second order are

$$\frac{d^2x}{ds^2} + (1-n)h^2x = h\delta + h^2(v_R-n')y \tag{10}$$
$$+ h^3(2n-\beta-1)x^2 + h^3(2\beta'-3n'+v_R)xy$$
$$+ \frac{1}{2}h^3(2\beta-n)y^2 + \frac{1}{2}hx'^2 - \frac{1}{2}hy'^2$$
$$+ h^2(2-n)x\delta - h^2(v_R-n')y\delta - h\delta^2$$

$$\frac{d^2y}{ds^2} + nh^2y = v_Rh - v_Rh\delta + h^2(2v_R-n')x$$
$$+ h^3(\beta'-2n'+v_R)x^2 + 2h^3(\beta-n)xy$$
$$- \frac{1}{2}h^3(2\beta'-n'-v_R)y^2 - h^2(2v_R-n')x\delta$$
$$+ h^2ny\delta + \frac{1}{2}v_Rhx'^2 + hx'y'$$
$$+ \frac{1}{2}v_Rhy^2 + v_Rh\delta^2$$

Second-order non-midplane-symmetric matrix elements can arise from three different sources:

1) Second-order non-midplane-symmetric terms coupling to midplane symmetric first-order matrix elements

2) Second-order midplane-symmetric terms coupling to non-midplane-symmetric first-order matrix elements.

3) First-order non-midplane-symmetric terms coupling to midplane symmetric second-order matrix elements.

When second-order terms are included, the shift in the central trajectory due to the vertically bending field causes a change in the first-order terms.

Space limitations prohibit a listing of the non-midplane-symmetric second-order matrix elements, or of the alterations to the first-order terms. They involve three times as many terms as the midplane-symmetric second-order terms. The effects of the non-midplane-symmetric magnetic field components have been included in the computer programs TRANSPORT[1] and TURTLE[2].

1) K. L. Brown, F. Rothacker, D. C. Carey, and Ch. Iselin, TRANSPORT, A Computer Program for Designing Charged Particle Beam Transport Systems, SLAC Report No. 91, Fermilab Report No. 91, CERN 80-04.

2) D. C. Carey, TURTLE (Trace Unlimited Rays Through Lumped Elements), A Computer Program For Simulating Charged Particle Beam Transport Systems, Fermilab Report No. 64.